HLAA-2 Math Summer Review

Welcome to HLAA Year 2!

Often over the summer it is easy to forget some of the things you have learned. Here at Calverton we like to send work home to be completed throughout the summer in order to help students start their next year off strong.

Please compete the attached worksheets throughout the summer and avoid completing them all in the week before school starts. Please make sure you show all your work along the way.

Please read the two IA's, annotate by marking where you see or do not see the criteria, and fill out the grading sheet with reasons for each value.

The worksheet and the IA's will count as your first grades and needs to be completed for the first day of school.

There is a math help day for support in completing the required summer work.

Please email Mrs. Popernack, apopernack@calvertonschool.org, for any questions about the requirements.

Have a great summer!

Question 1

Use the extension of the binomial theorem for $n \in \mathbb{Q}$ to show that $\sqrt{\dfrac{1+x}{1-x}} \approx 1 + x + \dfrac{x^2}{2}$, $|x| < 1$.

Question 2

Using mathematical induction, prove that $1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{Z}^+$.

Question 3

(a) Show that $(2n-1)^3 + (2n+1)^3 = 16n^3 + 12n$ for $n \in \mathbb{Z}$. [3]

(b) Hence, or otherwise, prove that the sum of the cubes of any two consecutive odd integers is divisible by four. [3]

Question 4

Expand $(2x - 3)^4$ in descending powers of x and simplify your answer.

Question 5

Let $f(x) = 2x$, $g(x) = 4x + 6$ and $h(x) = (f \circ g)(x)$, for $x \in \mathbb{R}$.

(a) Find $h(x)$. [2]

(b) Find $h^{-1}(x)$. [3]

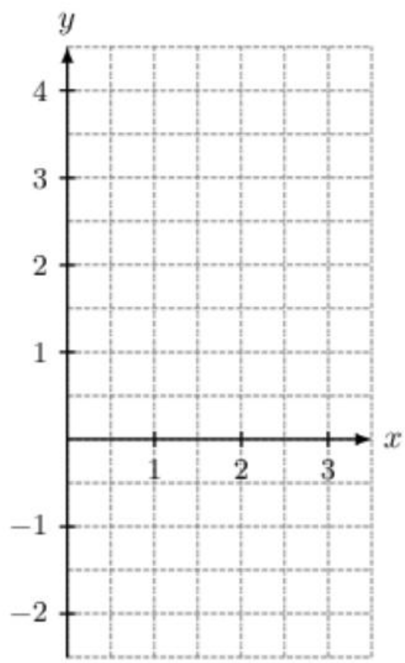
Question 6

[Maximum mark: 8]

Let $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x^2 - 4$, for $x \in \mathbb{R}$.

(a) Show that $(f \circ g)(x) = \frac{1}{4}x^4 - 2x^2 + 2$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$, for $0 \leq x \leq 3$. [3]



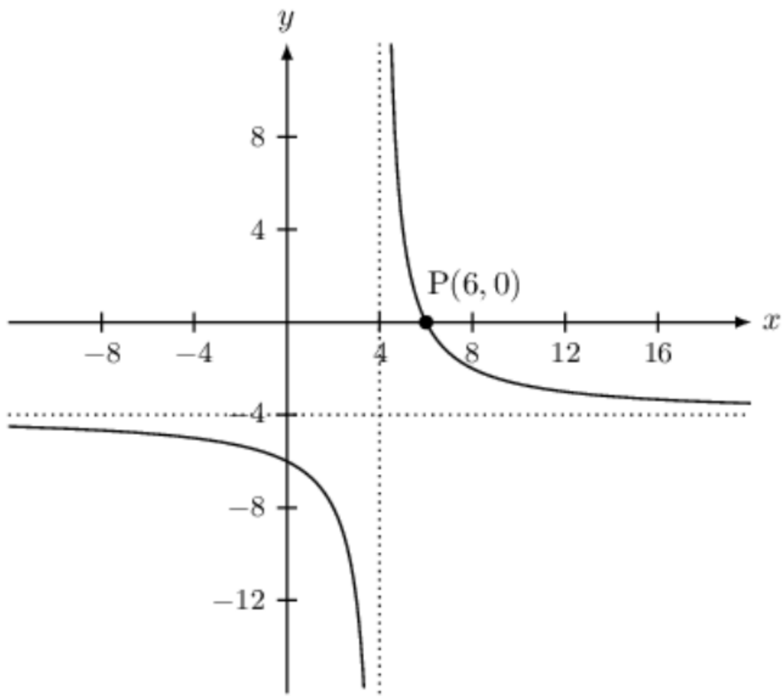
(c) The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \leq x \leq 3$. Find the possible values of k. [3]

Question 7

[Maximum mark: 4]

A rational function is defined by $f(x) = a + \dfrac{b}{x - c}$, for $x \neq c$, where $a, b, c \in \mathbb{Z}$.

The following diagram represents the graph of $y = f(x)$.



Using the information on the graph,

(a) state the value of a and the value of c ; [2]

(b) find the value of b. [2]

Question 8

Let $f(x) = 2x^2 - 5x + 7$. The line L intersects f at $\mathrm{P}(3, 10)$ and is perpendicular to the tangent to the curv of f at P. Find the equation of L in the form $y = mx + c$.

Question 9

Consider the function $f(x) = \ln(2x - 1)$. Let point A be the point on the curve where $x = 3$.

(a) Write down the gradient of the curve at A. [2

(b) The normal to the curve at A cuts the x-axis at P. find the coordinates of P. [5

Question 10

Find $\frac{dy}{dx}$ from First Principles: $y = 5x^3$

Question 11

Note: In this question, distance is in metres and time is in seconds.

A tennis ball is thrown in the air. Its height h above the ground after time t is given by

$$h(t) = -5t^2 + 20t + 4, \text{ for } 0 \leq t \leq 4.$$

(a) Find $h'(t)$. [2]

(b) Find the maximum height attained by the ball. [3]

Question 12

The curve C is defined by the equation $x^2y + \ln(xy) = 1$, $x > 0$, $y > 0$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Determine the equation of the tangent to C at the point $\mathrm{P}(1, 1)$. [3]

Question 13

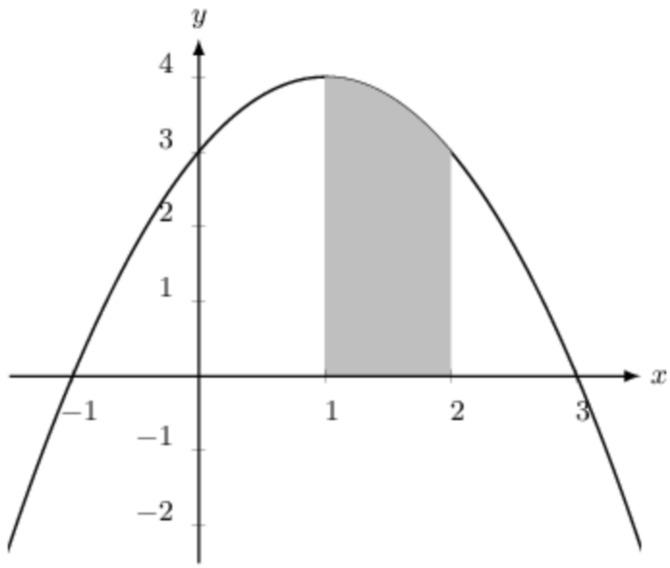
Air is being pumped into a spherical balloon so that its volume is increasing at a constant rate of 15 $\text{cm}^3\,\text{min}^{-1}$.

The surface area S and the volume V of a sphere of radius r are given by $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$.

Find the rate at which the surface area of the balloon is increasing when its radius hits 10 cm.

Question 14

Let $f(x) = -x^2 + 2x + 3$. The graph of f is shown in the following diagram.



(a) Find $\int (-x^2 + 2x + 3)\, \mathrm{d}x$. [2]

(b) Find the area of the shaded region. [3]

Question 15

The function $f(x)$ lies entirely to the right of the $y-$axis, it has no $y-$intercept and it has a derivative of $f'(x) = \frac{x^3}{\sqrt{x^4 - 27}}$.

(a) (i) Show that $f''(x) = \frac{x^2(x^4 - 81)}{\sqrt{(x^4 - 27)^3}}$.

(ii) Hence justify that f has a single point of inflection when $x = 3$. [6]

(b) Find $\int \frac{x^3}{\sqrt{x^4 - 27}} \, dx$. [3]

It is known that $\int_3^6 f(x) \, dx = 18.33$, to four significant figures.

(c) Find $f(x)$. [3]

Question 16

Solve the equation $\log_2(x^2 - 2x + 1) = 1 + \log_2(x - 1)$.

Question 17

Find the value of each of the following, giving your answer as an integer.

(a) $\log_{10} 100$. [2]

(b) $\log_{10} 50 + \log_{10} 2$. [2]

(c) $\log_{10} 4 - \log_{10} 40$. [3]

Question 18

Let $z = 2 + \mathrm{i}$ and $w = 1 - 2\mathrm{i}$.

(a) Find zw. [2]

(b) Illustrate z, w and zw on the same Argand diagram. [3]

(c) Let θ be the angle between zw and w. Find θ, giving your answer in radians. [3]